Séminaire Itinérant Géométrie et Physique II

July 27 - July 30, 2004
Morningside Mathematics Centre
Beijing, China

Tuesday July 27

14:00 – 14:50	Edward FRENKEL
15:15 – 16:05	Olivier MATHIEU
16:30 – 17:20	Kyoji SAITO

Wednesday July 28

09:00 – 09:50	Pierre SCHAPIRA
10:10 – 11:00	Edward FRENKEL
11:20 – 12:10	Olivier MATHIEU

Thursday July 29

09:00 – 09:50	Kyoji SAITO
10:10 – 11:00	Pierre SCHAPIRA
11:20 – 12:10	Edward FRENKEL
15:10 – 16:00	Feng Xu
16:20 – 17:10	Andrea D'AGNOLO

Friday July 30

09:00 – 09:50	Olivier MATHIEU
10:10 – 11:00	Kyoji SAITO
11:20 – 12:10	Pierre SCHAPIRA

Geometric Langlands correspondence and Kac-Moody algebras

Edward FRENKEL
University of California, Berkeley, USA

The geometric Langlands conjectures predict that one can associate to any local system on a complex algebraic curve X a D-modules on the moduli spaces of bundles on X. This D-module has to be a "Hecke eigensheaf" with the "eigenvalue" being the local system. I will describe a possible way to contruct Hecke eigensheaves using representations of affine Kac-Moody algebras. This construction is motivated by and is a direct generalization of the construction of the sheaves of conformal blocks in the Wess-Zumino-Witten model of conformal field theory. In genus zero these D-modules correspond to the differential equations of the Gaudin model.
References: math.AG/0303074, q-alg/9506003

Connections on stable bundles

Olivier MATHIEU
Institut Girard Desargues, Lyon, France

Let X be a Riemmann surface, i.e a genus g surface with a complex structure, and let Γ be its fundamental group. To each n-dimensional representation $\rho : \Gamma \to GL(n, \mathbf{C})$ corresponds a rank n holomorphic bundle $E(\rho)$ on X. Its total space is $(\tilde{X} \times \mathbf{C}^n)/\Gamma$, where $\tilde{X}$ is the universal cover of X, where Γ acts diagonally on $\tilde{X} \times \mathbf{C}^n$ and where the action on the second component $\mathbf{C}^n$ is determined by ρ. It is natural to ask: *Among all holomorphic vector bundles on X, which are those isomorphic to $E \simeq E(\rho)$ for some $\rho : \Gamma \to GL(n, \mathbf{C})$?*. Using the Riemann-Hilbert correspondance, it is equivalent to the following question: *When does E admit an holomorphic connection?*.

A. Weil gave a complete answer to this question. Since the curvature of any holomorphic connection vanishes, its first Chern class $c_1(E)$ vanishes. Conversely, A. Weil has shown that if E is indecomposable (i.e. is not a direct sum) and if $c_1(E) = 0$, then E admits an holomorphic connection.

However this holomorphic connection is not unique. In Weil's statement, the space of holomorphic connection is an affine space of dimension $\geq 1 + (g-1)n^2$. The next question is: *Out of these connections, could we single out one nicest connection?*

This question has been answered by Narashiman and Seshadri **[NS1][NS2]**. Their solution requires the notion of stability, which is now recalled. Indeed we will use the terminology "stable" instead of "stable of slope 0". An holomorphic vector bundle E is called *stable* if $c_1(E) = 0$, but $c_1(F) < 0$ for any proper subbundle

F. Here the meaning of the condition "$c_1(F) < 0$" is explained by the natural identification $H^2(X) \simeq \mathbf{Z}$. Narashiman and Seshadri theorem states that a stable bundle E admits a unique hermitian holomorphic connection, i.e. E is isomorphic to $E(\rho)$ for a unique unitarisable representation ρ (i.e. $\overline{\mathrm{Im}\,\rho}$ is a compact subgroup of $GL(n, \mathbf{C})$).

In order to raise our last question, we need some new hypotheses. Let K be a number field, let X be a complete curve of genus g and let E be a stable vector bundle. The notion of stability is defined in this context, using the degree instead of the first Chern class. Obviously, we can reformulate Narashiman and Seshadri theorem: for each embedding $\sigma : K \to \mathbf{C}$, there is a unique hermitian connection on the bundle $\mathbf{C} \otimes E$ over $X \times_K \mathbf{C}$. Therefore to each infinite place of K is attached a certain connection. *What about finite places of K?*

In the talks, we will explain the result obtained in **[M]**, which is an answer to the last question Also, a conjecture about the algebraicity of the solutions of certain differential equations is stated.

Bibliography:

[M]: O. Mathieu: Connections on stable bundles, In preparation
[NS1]: M.S. Narasimhan and C.S. Seshadri: Holomorphic vector bundles on a compact Riemann surface. Math. Ann. 155 (1964) 69-80.
[NS2]: M.S. Narasimhan and C.S. Seshadri: Stable and unitary vector bundles on a compact Riemann surface. Ann. of Math. 82 (1965) 540-567.

The geometry of finite reflection groups

Kyoji Saito
RIMS, Kyoto, Japan

1. Flat structure on the quotient space of a finite reflection group
2. Adjoint quotient morphism for simple Lie algebra and the primitive form
3. The braid groups and the $K(\pi, 1)$-space

Abstract: Finite reflection group W and the quotient variety V/W for a vector representation V of W is a simple geometric object, where the interests of several different aria of mathematics, e.g. Lie and representation theory, complex and differential geometry, analysis and integrable systems,... are intersecting. The study of this (finite reflection group) case is often the proto-type of further developpment of the theory. So, in this lecture, I want to concentrate only in this case. Even in such, in its appearance, simple cases, there are many deep unsolved questions which seems to lead to certain newest subject: the (Drinfeld) associators.

Finiteness and index theorems for elliptic pairs and for modules over quantization-deformation rings

Pierre SCHAPIRA
University of Paris 6, Paris, France

An elliptic pair on a complex manifold X is the data of a coherent module $\mathcal{M}$ over the ring $\mathcal{D}$ of differential operators and an R-constructible sheaf F on X, the characteristic variety of $\mathcal{M}$ not intersecting the microsupport of F outside the zero-section of the cotangent bundle T^*X. If the support of the pair is compact, the complex of solutions of $\mathcal{M}$ with values in the sheaf of generalized holomorphic functions associated with F has finite dimensional cohomology over C, and the index is calculated in terms of the Euler class of $\mathcal{M}$ and that of F. This result gives a new approach and a generalization of many classical ones, including the Atiyah-Singer theorem.

Next, we consider the case of complex symplectic manifolds. The ring $\mathcal{D}$ is replaced with a quantization-deformation ring $\mathcal{W}$ (which may not exist globally, but the stack of $\mathcal{W}$-modules exists) and the finiteness and index theorems (partly conjectural) hold when replacing C with a field K containing a parameter $\hbar$.